

# OPTIMIZATION OF WATER REMOVAL IN THE PRESS SECTION OF A PAPER MACHINE

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**Abstract** - An optimization problem regarding water removal in the press section of a paper machine is considered in this work. The proposed model tries to minimize a cost function comprising the replacement of the felts in the press section, the cost of energy to operate the press and the cost of energy in the drying section, while satisfying the constraints of water mass balance in the process. The model is classified as a mixed-integer nonlinear program (MINLP) in which the most important decisions are: a) the sequence of paper to produce or when to produce the paper; b) the need to exchange the felts; and c) when to exchange the felts. Numerical examples are presented to illustrate the performance of the model.

*Keywords:* Optimization; Schedule; GAMS; MINLP.

## INTRODUCTION

With increasing interest in optimizing parts of paper machines in recent years, there has been much work addressing the solution of these problems. Most of the investigations that have been reported have focused on different parts of a paper mill, for example on paper-converting (Westerlund et al., 1980). Also, several authors have studied water removal in the press section, as investigated by Wahlstrom (1960), who reported the water removal by a vertical flow at the NIP (contacting zone between two rolls). Different mathematical models for this part of a paper machine are reported by Kerekes et al. (1991).

The paper manufacturing process is basically the same for any plant. Initially the mass of paper is prepared; it goes to the formation section, and then to the press section for water removal, and finally to the drying section. In a paper machine, specifically in the press section, there are two important points: the

felts, whose function is to carry the wet web through the press nip (Farouk, 1991), and the NIP, which is the contacting zone between two rolls, where the water is transferred from the web to the felt. In the press section, which is the object of this work, the average life of a felt depends on its type, ranging from 35-45 days.

The press section is an important part of the machine, affecting the properties of the paper, as well as having an impact on the manufacturing cost. Low efficiency in this section causes difficulties, like reduction of the tensile strength, increase of the steam consumption in the drying section of the machine and, in many cases, the loss of productivity due to the reduction of the speed of the machine. According to Reese (2006b), it is more economical to squeeze water from the web in the press section than to evaporate water in the drying section. A reduction of 1% humidity in the leaf usually results in a reduction of steam consumption of around 4.5% in

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the drying process (Figure 1). Recently, there has been significant improvement in the operation of presses to improve economy by increasing the water removal in the press section and, at the same time, keeping or improving the characteristics of the web of paper.

This work considers the novelty of minimizing the replacement of the felts, by using an optimal sequence of paper production, in order to improve the water removal in the press section. The main goal is to optimize the change of felts in a paper machine, while determining the optimal sequence of the production for different types of paper, the felts to be replaced, and the optimal cost of production. The objective for optimizing the press section is to obtain the best sequence of production of the paper reels and the time of replacement of felts, increasing the water removal in the press section. In this way, less water needs to be removed in the drying section, where the energy costs are higher. A comparison of the model results with actual industrial values is presented.

## PAPER MACHINE DESCRIPTION

Until the 18<sup>th</sup> century, paper sheets were made manually. The laborer immersed a screen, attached to a wood frame in a tank containing a fiber suspension, forming a leaf that was subsequently dried in air. In 1799, Nicolas Robert invented a machine that made possible the formation of a sheet of paper of infinite length. It was made of wood and had a suspended fabric screen, in which the fiber suspension was sparged. Robert, due to financial and technical difficulties, could not go ahead with his project and was obliged to sell the patent to the Fourdrinier brothers, who did not have much success as well. In 1818, Donkin successfully built, in Germany, the first machine of this type. In it, the mass remained in agitation and fell on the screen by gravity. After being pressed, the leaf was rolled up at

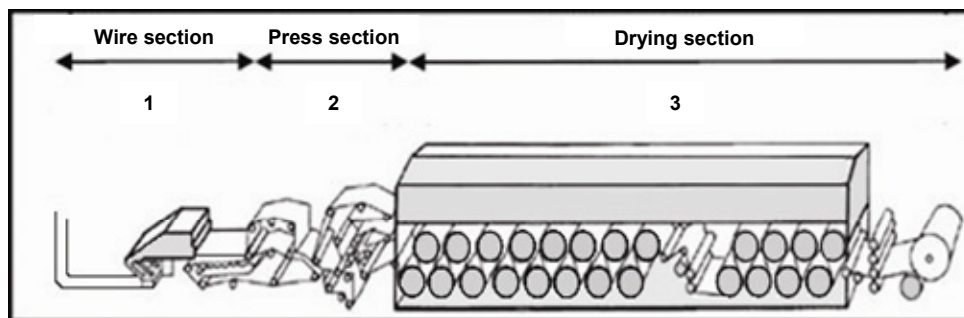
the end of the machine. The modern machines, based on the one invented by Robert, are constituted of many independent sections, each one with its function and proper characteristics. Almost all the sections of a paper machine can be remodeled and improved, with the exception of the width that is fixed. In general, the parts of a machine to manufacture paper are: the wire section, the press section and the drying section (Figure 1). The removal of water starts by gravity, is followed by suction and pressing, and finishes by evaporation. Currently, due to increasing energy costs, efforts are concentrated on the drying.

The amount of water removed in each section varies according to the kind of paper used, the operating conditions, and the machine settings. In general, as shown in Figure 1, the wire section reduces the amount of water from 99% to around 80%, the press section reduces it from around 80% to around 50%, and the drying section reduces it from around 50% to 5%.

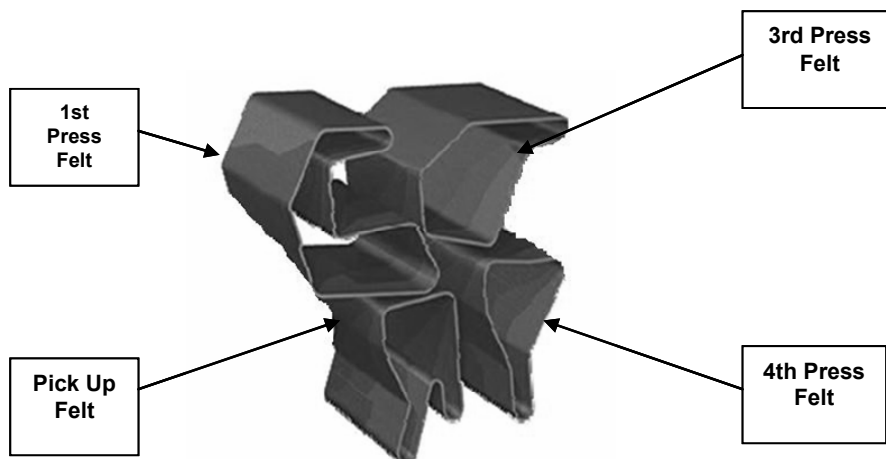
## PRESS SECTION AND FELTS

The high replacement frequency of the felts in the press section has received increased attention in the last few years, considering that they are very expensive. Also, the amount of water that can be removed by pressing the paper before it enters the drying section embodies the largest potential savings in the total cost of water removal. The drying section is responsible for around 78% of the cost of water removal in the web of paper, while about 10% of the cost is in the wire section and about 12% of the cost is in the press section. These values depend on both the unitary costs in each section and the amount of water removed in each one.

The press section is shown in Figure 2, in which four different felts are shown.



**Figure 1:** Sections of a paper machine (with paper humidity, given as mass water/mass cellulose): entering the wire section (99%), entering the press section (80%), entering the drying section (50%) and leaving the paper machine (5%).



**Figure 2:** Press section.

Felts are responsible for guiding the web, removing the maximum amount of water, smoothing the surface, and removing small marks left in the formation section (Reese, 2006a). The felts have different sizes, costs and physical characteristics. The felts lifetime depends on the amount of water they remove during their useful life. According to Roux and Vincent (1991), the water removal by pressing is usually 20 times cheaper than by drying.

**PROBLEM STATEMENT**

The objective is to minimize the total cost, which involves the cost of new felts (replacement), the cost of pressing and the cost of steam for the drying section. The assumptions are a fixed time horizon, different types and amount of paper that can be processed in the same paper machine, and the use of new felts at the start of the production.

**MATHEMATICAL MODEL**

Initially the amount of reels to be produced is defined. Each set of reels contains a specific type of paper. The total production time  $T$  is given by:

$$T = \sum_{i=1}^N T_i \tag{1}$$

in which  $T_i$  is the processing time for each set of reels.

In this work the total time did not regard shutdowns due to unexpected events, such as rupture

of the paper. A variable for the reels is then defined as follows:

$$x_{ik} = \begin{cases} 1 & \text{for reel } i \text{ processed in time interval } k \\ 0 & \text{otherwise} \end{cases}$$

where each time interval does not necessarily have the same time length, since it depends on the processing time of each reel. The following constraints then hold: each reel (or set of reels) can only be processed in one interval; each interval can only process one reel (or set of reels). These assignment constraints are given by:

$$\sum_{k=1}^N x_{ik} = 1 \quad i = 1, \dots, N \tag{2}$$

$$\sum_{i=1}^N x_{ik} = 1 \quad k = 1, \dots, N \tag{3}$$

Therefore, the time for each interval is given by:

$$t_k = \sum_{i=1}^N T_i \cdot x_{ik} \tag{4}$$

in which these variables are related to the total time  $T$  as follows:

$$T = \sum_{k=1}^N t_k \tag{5}$$

Considering that, at the beginning, all felts are new, a binary variable  $y_{jk}$  is defined to represent the potential replacement of a felt in position  $j$  (press  $j$ ) at the beginning of interval  $k$  (Figure 3). The positions in the press machine are shown as in Figure 2 (there are four of them). The binary variable for replacement of felts is defined as follows:

$$y_{jk} = \begin{cases} 1 & \text{if the felt in position } j \text{ was changed at the} \\ & \text{beginning of time interval } k \\ 0 & \text{otherwise} \end{cases}$$

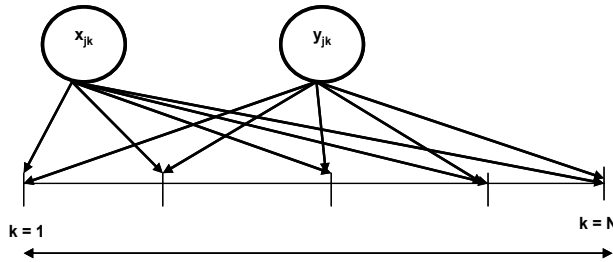


Figure 3: Binary Variables.

Since the production starts with new felts, at the beginning of the first time interval,  $k = 1$ :

$$y_{j1} = 1 \quad j = 1, 2, 3, 4 \quad (6)$$

In this model, the objective function includes the cost of new felts, the cost of energy in the press section, and the cost of energy in the drying section. Since the total time is fixed, the objective function uses the mass of water removed from the reels in the drying section at each interval  $k$ ,  $m_k$ , and not the mass flow rate,  $\dot{m}_k$ . The objective function is then given by:

$$\min z = \sum_{k=1}^N \sum_{j=1}^4 CF_j \cdot y_{jk} + \sum_{k=1}^N \sum_{i=1}^N CP_{ik} \cdot x_{ik} + \sum_{k=1}^N cs_k \cdot m_k \quad (7)$$

The unitary costs  $CF_j$  and  $CP_{ik}$  are parameters in the optimization (given values) and so the first and second terms on the right hand side of Equation (7) are linear. However, the third term is nonlinear and far more complicated. The cost of energy for drying in the interval  $k$  is given by:

$$cs_k = \sum_{i=1}^N CS_{ik} \cdot x_{ik} \quad (8)$$

The mass of water evaporated in the interval  $k$  in the drying section is given by:

$$m_k = m_k^e - m_k^s - m_k^{rp} \quad (9)$$

$$m_k^e = \sum_{i=1}^N [U_i^e \cdot A_i] \cdot x_{ik} \quad (10)$$

$$m_k^s = \sum_{i=1}^N [U_i^s \cdot A_i] \cdot x_{ik} \quad (11)$$

The water balance in Equation (9) corresponds to the mass of evaporated water in interval  $k$ ; the variable  $m_k^e$  is the initial amount of water in the press section;  $m_k^s$  is the amount of water at the end of the drying section, and  $m_k^{rp}$  is the amount of water removed by pressing (Figure 4). The values of the area of the processed reels ( $A_i$ ) and the humidity of the paper per unit of area at the entrance and exit of the process ( $U_i^e$  and  $U_i^s$ ) are known.

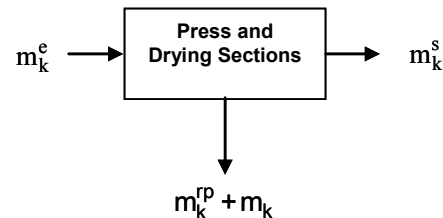


Figure 4: Water balance.

The mass removed by pressing is given by the summation of the amount of water removed in the four felts:

$$m_k^{rp} = \sum_{j=1}^4 m_{jk} \quad k = 1, \dots, N \quad (12)$$

The water removed in the press section is also equal to the mass of water in the initial part of the press section minus the water left in the paper after leaving the press section (and before entering the drying section):

$$m_k^{sp} = m_k^e - m_k^{sp} \quad k = 1, \dots, N \quad (13)$$

Following the model of water removal during the pressing developed in this work (see Appendix 1), the amount of water at the end of the press section, at interval  $k$ , is given by Equation (14):

$$m_k^{sp} = m_k^e \cdot \prod_{j=1}^4 \frac{1}{[1 + \bar{\Lambda}_{jk}]} \quad k = 1, \dots, N \quad (14)$$

Here the extraction factor  $\bar{\Lambda}_{jk}$  depends on the average time utilization of the felt  $\bar{\tau}_{jk}$  by the following expression,

$$\bar{\Lambda}_{jk} = \Lambda_{jk}^0 \cdot \frac{\xi_{0,j} \cdot \exp(-\alpha_j \cdot \bar{\tau}_{jk})}{[1 - (1 - \xi_{0,j}) \cdot \exp(-\alpha_j \cdot \bar{\tau}_{jk})]} \quad (15)$$

$$\Lambda_{jk}^0 = \sum_{i=1}^N \lambda_{ij} \cdot x_{ik} \quad (16)$$

in which  $\lambda_{ij}$  is the extraction factor of a new felt  $j$  for each reel  $i$ ,  $\alpha_j$  is the coefficient of reduction of the useful life of the felt, and  $\xi_{0,j}$  is the volumetric fraction of fiber in the new felt. For the complete derivation of this equation, see Appendix 1.

In Equation (15), it is possible to observe that when  $\bar{\tau}_{jk} \rightarrow \infty$ ,  $\bar{\Lambda}_{jk} \rightarrow 0$ , which in Equation (14) implies that  $m_k^{sp} \rightarrow m_k^e$ , the felt is not able to remove more water from the paper.

The average time of the felt running in the machine,  $\bar{\tau}_{jk}$ , is equal to the time of felt  $j$  at the beginning of interval  $k$ ,  $\theta_{jk}$ , plus the average length of this time interval,  $t_k$ , as given by Equation (17).

$$\bar{\tau}_{jk} = \theta_{jk} + 0.5 \cdot t_k \quad (17)$$

The value of  $\theta_{jk}$  depends on whether the felt is the same one running in the machine ( $y_{jk} = 0$ ), or if it is a new felt after replacement ( $y_{jk} = 1$ ). The variable  $\theta_{jk}$  measures the running time of the felt since the last replacement. Therefore, for a new felt  $\theta_{jk} = 0$ . The running times of operation of the felt in the press in the interval  $k$  are then given by:

$$\tau_{jk} = \theta_{jk} + t_k \quad k = 1, \dots, N \quad (18)$$

$$\theta_{jk} = \tau_{j,k-1} \cdot (1 - y_{jk}) \quad k = 2, \dots, N \quad (19)$$

$$\theta_{j1} = 0 \quad (20)$$

In this model, the time required to change a felt was neglected because it typically represents only 1%-3% of the total time. The operating cost of changing a felt can be included in the cost of purchasing a new one in the value of the parameter  $CF_j$ .

The model given by Equations (2) – (4) and (6) – (20) corresponds to a mixed-integer nonlinear program (MINLP) that involves the product of integer and continuous variables, such as in Equations (7), (15) and (16), and (19), which is not a suitable form to use in an optimization model. To obtain a more tractable model, the restrictions are changed into an equivalent model with linear terms containing only integer variables and nonlinear terms containing only continuous variables.

First, constraint (19) can be replaced by the equivalent set of constraints,

$$\theta_{jk} = \tau_{j,k-1}^{(2)} \quad (21)$$

$$\tau_{j,k-1} = \tau_{j,k-1}^{(1)} + \tau_{j,k-1}^{(2)} \quad (22)$$

$$0 \leq \tau_{j,k-1}^{(1)} \leq M_1 \cdot y_{jk} \quad (23)$$

$$0 \leq \tau_{j,k-1}^{(2)} \leq M_1 \cdot (1 - y_{jk}) \quad (24)$$

in which  $M_1$  is a parameter that is sufficiently large so as to include all possible values of all previous times.

Second, the term  $cs_k \cdot m_k$  is replaced in Equation (7) by:

$$\begin{aligned} cs_k \cdot m_k &= cs_k \cdot (m_k^e - m_k^s - m_k^{sp}) = \\ cs_k \cdot (m_k^e - m_k^s - m_k^e + m_k^{sp}) &= \\ cs_k \cdot m_k^{sp} - cs_k \cdot m_k^s & \end{aligned} \quad (25)$$

The last term in Equation (25) can be rewritten as,

$$cs_k \cdot m_k^s = \left( \sum_{i=1}^N CS_{ik} \cdot x_{ik} \right) \cdot \left( \sum_{i=1}^N U_i^s \cdot A_i \cdot x_{ik} \right) = \sum_{i=1}^N CS_{ik} \cdot U_i^s \cdot A_i \cdot x_{ik} \quad (26)$$

since only one of the  $x_{ik}$  is equal to 1 in each summation.

The first term of the right hand side of Equation (25) can be rewritten as:

$$\sum_{k=1}^N cs_k \cdot m_k^{sp} = \sum_{i=1}^N \sum_{k=1}^N CS_{ik} \cdot ma_{ik} \quad (27)$$

$$m_k^{sp} = \sum_{i=1}^N ma_{ik} \quad (28)$$

in which  $ma_{ik} = 0$  if  $x_{ik} = 0$ . Equation (7) then becomes:

$$\begin{aligned} \min z = & \sum_{k=1}^N \sum_{j=1}^4 CF_j \cdot y_{jk} + \sum_{k=1}^N \sum_{i=1}^N CP_{i,k} \cdot x_{ik} + \\ & \sum_{k=1}^N \sum_{i=1}^N CS_{ik} \cdot ma_{ik} - \sum_{k=1}^N \sum_{i=1}^N CS_{ik} \cdot U_i^s \cdot A_i \cdot x_{ik} \end{aligned} \quad (29)$$

Third, the term of water extraction in the press section can be written as:

$$\ln(m_k^{sp}) = \ln(m_k^e) - \sum_{j=1}^4 \ln(1 + \bar{\Lambda}_{jk}) \quad (30)$$

$$\begin{aligned} \ln \bar{\Lambda}_{jk} = & \ln \Lambda_{jk}^0 - \alpha_j \cdot \bar{\tau}_{jk} + \ln(\xi_{0,j}) - \\ & \ln(1 - (1 - \xi_{0,j}) \cdot \exp(-\alpha_j \cdot \bar{\tau}_{jk})) \end{aligned} \quad (31)$$

Defining the auxiliary variables:

$$u_{jk} = \ln \bar{\Lambda}_{jk} \quad (32)$$

$$v_k = \ln(m_k^{sp}) \quad (33)$$

the following constraints then result in an equivalent model,

$$v_k = \sum_{i=1}^N (\ln(U_i^e \cdot A_i)) \cdot x_{ik} - \sum_{j=1}^4 \ln(1 + \exp(u_{jk})) \quad (34)$$

$$\begin{aligned} u_{jk} = & \sum_{i=1}^N (\ln \lambda_{ij}) \cdot x_{ik} - \alpha_j \cdot \bar{\tau}_{jk} + \ln \xi_{0,j} - \\ & \ln(1 - (1 - \xi_{0,j}) \cdot \exp(-\alpha_j \cdot \bar{\tau}_{jk})) \end{aligned} \quad (35)$$

$$ma_{ik} = m_{ik}^{(1)} \quad (36)$$

$$m_{ik}^{(1)} + m_{ik}^{(2)} = \exp(v_k) \quad (37)$$

$$0 \leq m_{ik}^{(1)} \leq M_2 \cdot x_{ik} \quad (38)$$

$$0 \leq m_{ik}^{(2)} \leq M_2 \cdot (1 - x_{ik}) \quad (39)$$

in which  $M_2$  is a parameter that is large enough to absorb the maximum amount of water that can be removed from reel  $i$ .

Note that the amount of water  $m_k^{sp}$  in the exit of the press section cannot be less than the amount of water  $m_k^s$  established for the end of the drying process. Therefore, the additional constraint must be added:

$$v_k \geq \sum_{i=1}^N (\ln(U_i^s \cdot A_i)) \cdot x_{ik} \quad (40)$$

The MINLP model is then formulated as the minimization of the objective function given by Equation (29), subject to the constraints given by Equations (2) – (4), (6), (17) – (18), (20) – (24), and (34) – (40).

## NUMERICAL SOLUTION

The proposed model, formulated as a mixed integer non-linear programming problem (MINLP), was solved using GAMS with the solver SBB. The GAMS (General Algebraic Modelling System) Software (Brooke et al, 1998) is a modelling system for solving mathematical programming problems, including linear (LP), nonlinear programming (NLP) and mixed-integer linear (MILP) and mixed-integer nonlinear programming problems (MINLP) problems. The models are specified in the form of algebraic equations involving continuous and/or integer variables. These models can be specified with indices, which allow the solution of large-scale problems. GAMS is also linked to a large variety of solvers, which correspond to software packages for solving LP, NLP, MILP and MINLP problems (see <http://www.gams.com>). Data can be supplied in the form of parameters and tables, and linked to Excel spreadsheets. Another important feature of GAMS is that it has exact differentiation capabilities, with

which it automatically generates the gradients for nonlinear optimization problems.

The particular solver that was used to solve the numerical examples was the software package SBB (Simple Branch and Bound) (Bussieck and Drud, <http://www.gams.com/presentations/or01/sbb.pdf>). SBB is based on the branch and bound method for solving MINLP problems (Grossmann, 2002). The branch and bound method consists of a tree search where, at each node, a relaxed NLP subproblem is solved in which all 0-1 variables are treated as continuous variables except a subset of them that are fixed to values of 0 or 1. An NLP with noninteger solution at a given node provides a lower bound (minimization case) for the corresponding descendant children nodes. An upper bound is obtained when a feasible integer solution is obtained at a given node. The search in the enumeration of the tree is terminated when the lower and upper bounds lie within a specified tolerance. The NLP software that was used in SBB was CONOPT (Drud, 1992), which is based on the reduced gradient method for NLP problems. The reduced gradient method is based on successively linearizing the constraints of an augmented Lagrangian reformulation of the NLP, where each subproblem is solved by projecting the gradient in a subspace of the linear constraints and by using a Quasi-Newton method for updating the Hessian matrix.

All case studies presented in the next section were run using GAMS 25.5 and CONOPT3 on a PC workstation with four Intel Xeon 3.2 GHz processors.

## CASE STUDIES

To illustrate the application and computational effectiveness of the proposed MINLP model, comparison of the model results with actual data

from industry are presented. In this work, 22 cases of felt exchange were considered, with different numbers of time intervals (from 3 to 8). The values of the data are from industrial practice in the related year and include demands and length of time horizon. Each case was solved with the proposed MINLP model. We first solved the model with a fixed sequence of the reel production (the same one used in the real industrial case), and next the same model allowed the selection of the optimal sequence. The results were then compared.

### Case Study 15

Since the amount of data is very large, we have chosen just one case study to detail. From the 22 cases considered, case study 15 was chosen and the data are presented in Tables 1 through 5. Data for other case studies can be found in Drummond (2008).

Table 1 presents the grammage of each reel of paper (cellulose mass per unit area), with data required in the model ( $U_i^e$ ,  $U_i^s$ ,  $A_i$ ,  $T_i$ ). Table 2 presents the data for the felts ( $CF_j$ ,  $\alpha_j$ ,  $\xi_{0,j}$ ). Table 3 presents the data for the extraction factor of a new felt for each reel ( $\lambda_{ij}$ ). Table 4 presents the energy costs in the press section ( $CP_{ik}$ ). Table 5 presents the energy costs in the drying section ( $CS_{ik}$ ).

Figure 5 show some results for case 15, in which the time horizon was 843 hours with 8 intervals for the 8 products. The size of the MINLP was 564 single equations, 548 single variables, 92 discrete variables, and typically required 206 min of CPU time in order to find the global optimum. The results for water removal, felt lifetime, felt exchange and reel production are presented in the next section.

**Table 1: Parameters of the paper in case study 15.**

| Paper (Reel) Parameters |                              |                               |                               |                         |           |
|-------------------------|------------------------------|-------------------------------|-------------------------------|-------------------------|-----------|
| Paper (i)               | Grammage (g/m <sup>2</sup> ) | $U_i^e$ (ton/m <sup>2</sup> ) | $U_i^s$ (ton/m <sup>2</sup> ) | $A_i$ (m <sup>2</sup> ) | $T_i$ (h) |
| P1                      | 56                           | 0.00004615                    | 0.000005                      | 79879.80                | 166       |
| P2                      | 60                           | 0.00004945                    | 0.000005                      | 20948.93                | 41        |
| P3                      | 63                           | 0.00005192                    | 0.000005                      | 66922.78                | 117       |
| P4                      | 70                           | 0.00005969                    | 0.000005                      | 39153.11                | 68        |
| P5                      | 75                           | 0.00006181                    | 0.000005                      | 188097.00               | 309       |
| P6                      | 80                           | 0.00006593                    | 0.000005                      | 29744.00                | 48        |
| P7                      | 90                           | 0.00007417                    | 0.000005                      | 57353.40                | 86        |
| P8                      | 105                          | 0.00008571                    | 0.000005                      | 5746.00                 | 8         |

**Table 2: Parameters of the felt in case study 15.**

| Felt position | Parameters of Felts |                     |             |
|---------------|---------------------|---------------------|-------------|
|               | $CF_j$<br>(US\$)    | $\alpha_j$<br>(1/h) | $\xi_{0,j}$ |
| PICK UP       | 82434.65            | 0.003750            | 0.6080      |
| 1ST PRESS     | 37197.97            | 0.001667            | 0.5843      |
| 3RD PRESS     | 40029.90            | 0.000833            | 0.6427      |
| 4TH PRESS     | 49484.69            | 0.002083            | 0.5800      |

**Table 3: Values of  $\lambda_{ij}$ .**

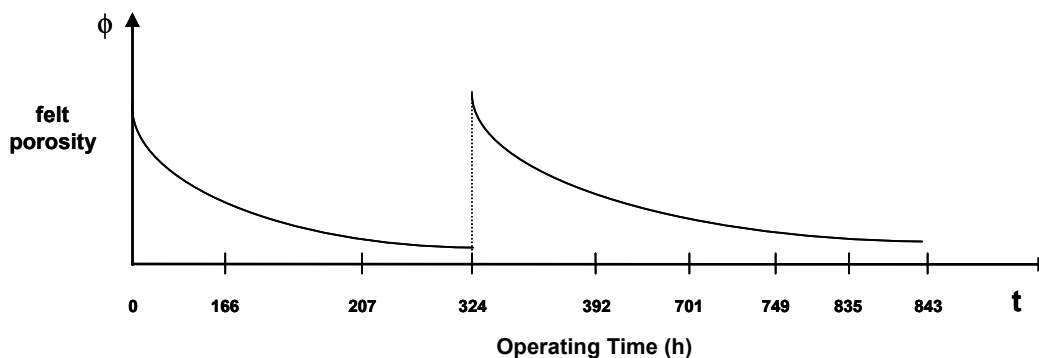
| Paper | Felt Position |           |           |           |
|-------|---------------|-----------|-----------|-----------|
|       | PICK UP       | 1ST PRESS | 3RD PRESS | 4TH PRESS |
| P1    | 0.2134        | 0.2813    | 0.1852    | 0.1250    |
| P2    | 0.2134        | 0.2813    | 0.1852    | 0.1250    |
| P3    | 0.2134        | 0.2813    | 0.1852    | 0.1250    |
| P4    | 0.2134        | 0.2813    | 0.1852    | 0.1250    |
| P5    | 0.2134        | 0.2813    | 0.1852    | 0.1250    |
| P6    | 0.2134        | 0.2813    | 0.1852    | 0.1250    |
| P7    | 0.2134        | 0.2813    | 0.1852    | 0.1250    |
| P8    | 0.2134        | 0.2813    | 0.1852    | 0.1250    |

**Table 4: Energy cost in the press section ( $CP_{ik}$ ) in US\$/ton for case study 15.**

| Paper | Time Intervals |        |        |        |        |        |        |        |
|-------|----------------|--------|--------|--------|--------|--------|--------|--------|
|       | T1             | T2     | T3     | T4     | T5     | T6     | T7     | T8     |
| P1    | 252.70         | 252.70 | 252.70 | 252.70 | 252.70 | 252.70 | 252.70 | 252.70 |
| P2    | 53.20          | 53.20  | 53.20  | 53.20  | 53.20  | 53.20  | 53.20  | 53.20  |
| P3    | 148.20         | 148.20 | 148.20 | 148.20 | 148.20 | 148.20 | 148.20 | 148.20 |
| P4    | 77.90          | 77.90  | 77.90  | 77.90  | 77.90  | 77.90  | 77.90  | 77.90  |
| P5    | 332.50         | 332.50 | 332.50 | 332.50 | 332.50 | 332.50 | 332.50 | 332.50 |
| P6    | 49.40          | 49.40  | 49.40  | 49.40  | 49.40  | 49.40  | 49.40  | 49.40  |
| P7    | 81.70          | 81.70  | 81.70  | 81.70  | 81.70  | 81.70  | 81.70  | 81.70  |
| P8    | 7.60           | 7.60   | 7.60   | 7.60   | 7.60   | 7.60   | 7.60   | 7.60   |

**Table 5: Energy cost in the drying section ( $CS_{ik}$ ) in US\$/ton for case study 15.**

| Paper | Time Intervals |          |          |          |          |          |          |          |
|-------|----------------|----------|----------|----------|----------|----------|----------|----------|
|       | T1             | T2       | T3       | T4       | T5       | T6       | T7       | T8       |
| P1    | 77044.90       | 77044.90 | 77044.90 | 77044.90 | 77044.90 | 77044.90 | 77044.90 | 77044.90 |
| P2    | 86881.09       | 86881.09 | 86881.09 | 86881.09 | 86881.09 | 86881.09 | 86881.09 | 86881.09 |
| P3    | 87919.16       | 87919.16 | 87919.16 | 87919.16 | 87919.16 | 87919.16 | 87919.16 | 87919.16 |
| P4    | 92591.40       | 92591.40 | 92591.40 | 92591.40 | 92591.40 | 92591.40 | 92591.40 | 92591.40 |
| P5    | 95851.16       | 95851.16 | 95851.16 | 95851.16 | 95851.16 | 95851.16 | 95851.16 | 95851.16 |
| P6    | 94439.02       | 94439.02 | 94439.02 | 94439.02 | 94439.02 | 94439.02 | 94439.02 | 94439.02 |
| P7    | 95894.12       | 95894.12 | 95894.12 | 95894.12 | 95894.12 | 95894.12 | 95894.12 | 95894.12 |
| P8    | 87446.04       | 87446.04 | 87446.04 | 87446.04 | 87446.04 | 87446.04 | 87446.04 | 87446.04 |

**Figure 5: Felt Porosity and Felt Exchange.**



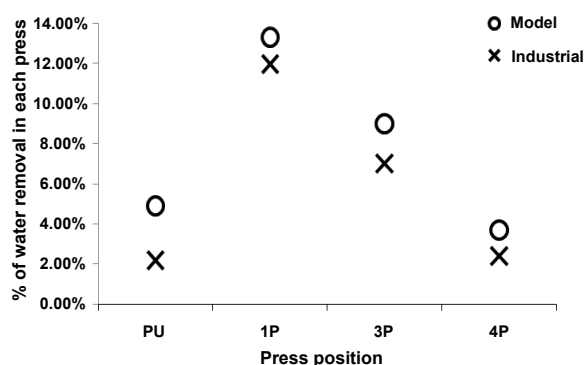


Figure 6: Comparing water removal in each press.

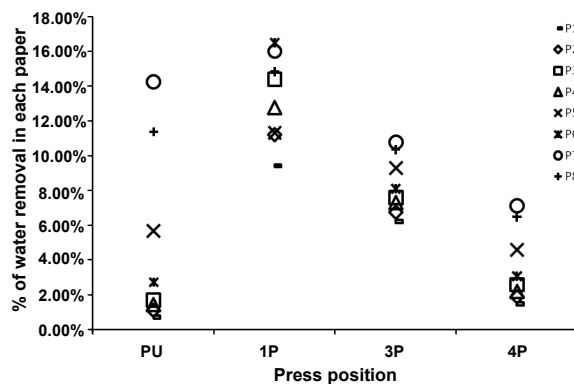


Figure 7: Comparing water removal of each paper in each press.

## NUMERICAL RESULTS

### Results of Case Study 15

In case study 15, a total of 528 reels were produced. Table 1 presents the type of papers produced for this case study.

In relation to the felt exchange in this case study, it was observed that we had an economy of 1.98% in relation to the cost of purchase and exchange of the felts, compared to the sequence used in industry and the optimized sequence found by the optimization of the model.

In Figure 6, we show the comparison between the results (industrial case x optimized sequence) in relation to the water removal in each press. In this case, we had an increase of 3.31% in the total water removal. In Figure 7, the results of water removal for each paper in each press are presented.

### Results for the Year

In this section we present the overall results for all 22 case studies considered, which amount to a one-year production in the industrial case. First the model was solved (optimized) with a fixed sequence of paper production, using the same one as in the actual industrial case; the model was then solved with a free sequence of paper production, using the same data. In both instances, the model determined the amount of water removed and the changing of felts. Each individual case study was solved separately, for both situations. The results were then compared.

It is important to point out that the optimized sequence and optimized felt exchange was not implemented in practice in the industrial case, so

these results only show the potential savings that could be obtained with the model.

### Water Removal

The challenge faced by the proposed mathematical model was the reduction in the replacement of felts through the selection of an optimal sequence of the reel production, increasing in this way the water removal in the press section and thereby reducing the cost of energy in the drying sector. Figure 8 presents the increase of water removal with the proposed model compared to the industrial data. This represents an annual increase of the order of 5% in the water removal, which is a significant result.

### Felt Lifetime

The lifetime of felts can also have a significant impact on the cost. In the proposed model, a fixed time horizon is used for processing the paper reels. However, the main issue in the replacement of the felts is to find a point where they can stay in the machine without losing their capacity for absorption. In this way, there is an increase in the potential useful lifetime of the felt, considering the improved efficiency in removing more water with the same felts.

With the proposed model, an average increase of 15% was obtained in the useful life of the felt by using the optimal sequence predicted from the mathematical model. Hence, besides the increase of the water removal, there is an increase of the useful life of the felt. Figure 9 presents the results obtained with the model in comparison with the industrial data.

## Felts Replacement

Replacement of felts is one aspect of the paper machine involving both cost and time loss. Replacing the felts requires stopping the machine for approximately 1 hour and 30 minutes and, after this, additional time is required to restart the machine. Therefore, a better sequence in the production of the reels minimized the exchanges of felts in the paper machine and, with this, resulted in a significant reduction in total cost and downtime of the machine. Figure 10 shows the exchanges in the four positions of each felt. A reduction

of 46% in the exchange of the felts is obtained, which also increases the lifetime of the felts.

## Cost Reduction

From the results, there is a total cost reduction in the operation of the plant. Table 6 indicates that the total cost was reduced by 4%. Some costs were not changed, like the costs in the press section, but the cost reduction in the drying section and the lower cost in the replacement of felts were the main reason for the total cost reduction.

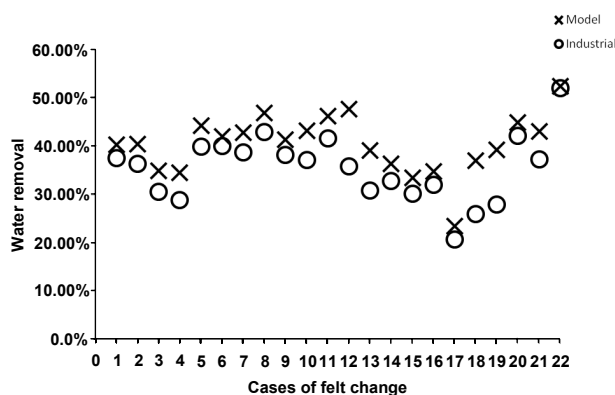


Figure 8: Water removal.

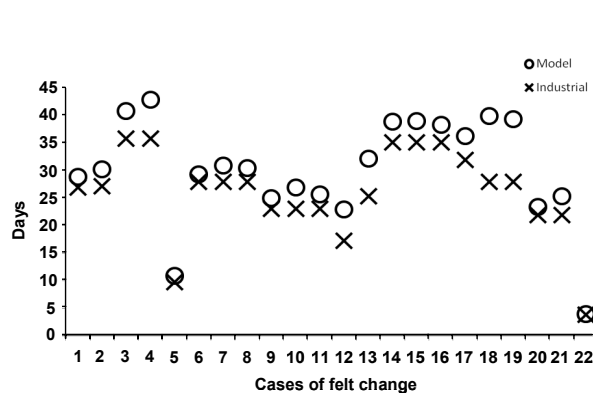


Figure 9: Felt lifetime.

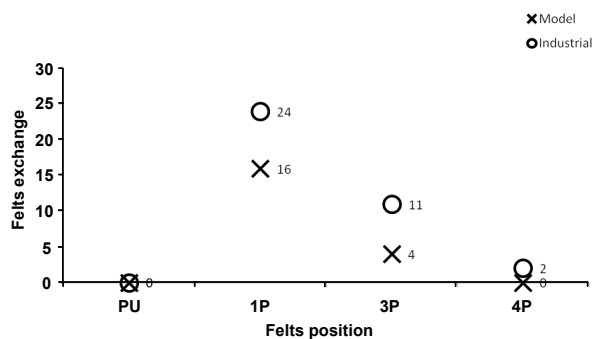


Figure 10: Difference between exchanges of felts.

Table 6: Costs for the optimized problem and the industrial case study.

| Cost   | Model (US\$)       | Industrial (US\$)  |
|--------|--------------------|--------------------|
| Felts  | US\$ 5,178,960.03  | US\$ 5,833,774.48  |
| Press  | US\$ 16,420.31     | US\$ 16,420.31     |
| Drying | US\$ 23,538,536.13 | US\$ 24,082,059.66 |
| Total  | US\$ 28,733,917.03 | US\$ 29,932,254.45 |

## CONCLUSIONS

In this work, an optimization model for paper production was proposed, in order to reduce operating costs and the replacement of felts. An MINLP model has been proposed to determine the replacement of felts in the press section of a paper machine, and the best sequence for processing the different kinds of paper scheduled for production.

It was shown that the problem could be efficiently solved, yielding better results than the ones from an industrial case. The improvement obtained in this the investigation considering 22 individual cases was a 5% increase in water removal in the press section, a 15% increase in felt lifetime and a 4% reduction in the total cost.

The proposed model could be used as a useful tool for planning the production of paper, once a fixed proposed production is selected. Also, by choosing the most appropriate time to change the felts, an increase in water removal can be achieved, reducing the costs.

## NOMENCLATURE

### Sets

|     |          |
|-----|----------|
| $i$ | reel     |
| $j$ | felt     |
| $k$ | interval |

### Variables

|            |  |
|------------|--|
| $m_k$      | amount of water removed in the drying section                          |
| $m_k^e$    | amount of water at the entrance of the press section in interval $k$   |
| $m_k^s$    | amount of water at the exit of the drying section in interval $k$      |
| $m_k^{rp}$ | amount of water removed by the pressing in interval $k$                |
| $m_k^{sp}$ | amount of water at the end of pressing in interval $k$                 |
| $ma_{ik}$  | mass of water at the exit of pressing of the reel $i$ in interval $k$  |
| $t_k$      | time of interval $k$   |
| $u_{jk}$   | auxiliary variable for the extraction coefficient of water in pressing |

|                   |  |
|-------------------|--|
| $v_k$             | auxiliary variable for water at the exit of the pressing in interval $k$ |
| $x_{ik}$          | reel $i$ processed in interval $k$                                       |
| $y_{jk}$          | felt in press $j$ exchanged at initial time $k$                          |
| $\tau_{jk}$       | time of life of the felt $j$ in interval $k$                             |
| $\bar{\tau}_{jk}$ | average of the time of the felt $j$ in interval $k$                      |
| $\theta_{jk}$     | initial time of the felt $j$ in interval $k$                             |

### Parameters

|                |   |
|----------------|---|
| $A_i$          | total area of the processed reels   |
| $CS_{ik}$      | steam cost in the drying section (\$/mass of water) in interval $k$             |
| $CF_j$         | cost of buying/changing (\$/felt)   |
| $CP_{ik}$      | energy cost expense in the press section of $i$ (\$/reel) in the interval $k$   |
| $M_1$          | big number (time)   |
| $M_2$          | big number (mass of water)  |
| $T_i$          | time to process reel $i$  |
| $U_i^e$        | humidity of the paper at the entrance of the press section (mass of water/area) |
| $U_i^s$        | humidity of the paper at the exit of the drying sector (mass of water/area)     |
| $\alpha_j$     | coefficient of reduction of the useful life of the felt                         |
| $\lambda_{ij}$ | extraction factor of the new felt $j$ with reel $i$ paper                       |
| $\xi_{0,j}$    | volumetric fraction of fiber in the new felt $j$                                |

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## APPENDIX 1

### Equations for the Dewatering In the Pressing Section

In this Appendix we present the development of the pressing equations used in this work. The mass of water that is removed during a certain interval  $t_k$  of time is:

$$m_k^{rp} = m_k^e - m_k^{sp} = \int_0^{t_k} (\dot{m}_k^e - \dot{m}_k^{sp}) \cdot dt \quad (A1)$$

where:

- $m_k^e$  mass water that entered the press section during time  $t_k$  in interval  $k$
- $\dot{m}_k^e$  outflow of water (inside of the paper) that enters the press section in interval  $k$
- $m_k^{sp}$  mass water that left the sector of pressing during time  $t_k$  in interval  $k$
- $\dot{m}_k^{sp}$  outflow of water (inside of the paper) that leaves the press section in interval  $k$

The water outflow at the entrance of the pressing process is given by:

$$\dot{m}_k^e = v_p \cdot L_p \cdot e_p \cdot \varepsilon_{H_2O}^e \cdot \rho_{H_2O} \quad (A2)$$

where:

- $v_p$  speed of the paper (m/s)
- $L_p$  width of the paper (m)
- $e_p$  thickness of the paper (m or mm/1000)

- $\varepsilon_{H_2O}^e$  volumetric fraction of water inside the paper at the entrance
- $\rho_{H_2O}$  density of water ( $\text{kg/m}^3$ )

Since these values remain constant at the entrance of the pressing for a given reel, we then have:

$$m_k^e = \int_0^{t_k} (v_p \cdot L_p \cdot e_p \cdot \varepsilon_{H_2O}^e \cdot \rho_{H_2O}) \cdot dt = v_p \cdot L_p \cdot e_p \cdot \varepsilon_{H_2O}^e \cdot \rho_{H_2O} \cdot t_k \quad (A3)$$

Since  $m_k^e$  is the mass of water inside of the reel at the entrance, we also have:

$$m_k^e = \sum_{i=1}^N A_i \cdot U_i^e \cdot x_{ik} \quad (A4)$$

where:

- $A_i$  total area of the reel ( $\text{m}^2$ )
- $U_i^e$  paper humidity at the entrance per unit area ( $\text{kg water/m}^2$ )

and:

$$A_i = (v_p \cdot T_i) \cdot L_p \quad (A5)$$

The water outflow inside the paper that leaves the press section in the interval  $k$  is given by:

$$\dot{m}_k^{sp} = v_p \cdot L_p \cdot e_p \cdot \varepsilon_{H_2O}^s \cdot \rho_{H_2O} \quad (A6)$$

where:

$\varepsilon_{\text{H}_2\text{O}}^s$  volumetric fraction of water inside the paper at the exit

The general balance of the volumetric fraction (cellulose, water and air) in each reel is given by:

$$\varepsilon_{\text{cel}} + \varepsilon_{\text{H}_2\text{O}} + \varepsilon_{\text{air}} = 1 \quad (\text{A7})$$

The porosity of the paper is the sum of the volumetric fractions of water and air:

$$\Phi_p = \varepsilon_{\text{H}_2\text{O}} + \varepsilon_{\text{air}} \quad (\text{A8})$$

These values change with the pressing. Hence, it is necessary to relate the volumetric fraction to the characteristics of the press and the felt used. On the other hand, it is considered that all of the reel properties (thickness, width) have the same values before and after the pressing (but not during the pressing; the thickness of paper change with the pressing, but it later returns to the previous value).

For each felt in each press, the general balance of volumetric fractions (fiber, water and air) is given by:

$$\xi_{\text{fiber}} + \xi_{\text{H}_2\text{O}} + \xi_{\text{air}} = 1 \quad (\text{A9})$$

The porosity of the felt is the sum of the volumetric fractions of water and air:

$$\phi = \xi_{\text{H}_2\text{O}} + \xi_{\text{air}} \quad (\text{A10})$$

It was found in this work that, from experimental analysis of industrial cases, the porosity of the felts decreased with the running time. The values of porosity were adequately fitted by an expression as follows, for each felt  $j$ :

$$\phi_j = \phi_{0,j} \cdot \exp(-\alpha_j \cdot t) \quad (\text{A11})$$

The mass fiber outflow of felt in each press machine  $j$  is given by:

$$\dot{m}_{\text{fiber}}^j = v_f \cdot L_f \cdot e_f \cdot \xi_{\text{fiber}} \cdot \rho_{\text{fiber}} \quad (\text{A12})$$

where:

$v_f$  speed of the felt (m/s)  
 $L_f$  width of the felt (m)  
 $e_f$  thickness of the felt (m or mm/1000)  
 $\xi_{\text{fiber}}$  volumetric fraction of fiber inside the felt  
 $\rho_{\text{fiber}}$  specific mass of the felt fiber ( $\text{kg/m}^3$ )

Each felt has its proper characteristics, but, for each felt, the outflow fiber mass that enters at the point of contact with the paper (nip) is equal to the outflow fiber mass that leaves it, since it is a closed circuit. Moreover, the total fiber mass in each felt is the same from the beginning to the end of its useful life. In this way we have:

$$\xi_{\text{fiber}} \cdot e_f = \text{constant} \quad (\text{A13})$$

Moreover, the relation between volumetric fraction of fiber and porosity in the felt is given by:

$$\xi_{\text{fiber},j} = 1 - \phi_j = 1 - \phi_{0,j} \cdot \exp(-\alpha_j \cdot t) \quad (\text{A14})$$

The thickness of each felt then varies with time as follows:

$$e_{f,j} = e_{f,j}^0 \cdot \frac{(1 - \phi_{0,j})}{(1 - \phi_{0,j} \cdot \exp(-\alpha_j \cdot t))} \quad (\text{A15})$$

In order to find the relation between the amount of water in the felt and the amount of water in the paper, a water balance is done, so that the sum of the water outflows (in the paper and in the felt) that enter the nip is equal to the sum of the water outflows that leave it.

Additionally, using the condition of equilibrium (with an equilibrium constant,  $k$ ), relating the ratios of volumetric fraction of water/porosity, in the paper and in the felt, we have that:

$$\frac{\xi_{\text{H}_2\text{O}}}{\xi_{\text{H}_2\text{O}} + \xi_{\text{air}}} = k \cdot \frac{\varepsilon_{\text{H}_2\text{O}}}{\varepsilon_{\text{H}_2\text{O}} + \varepsilon_{\text{air}}} \quad (\text{A16})$$

And, using the removal of water from the felt by the vacuum system of the pressing machine (vacuum removes a constant fraction of water,  $1 - \beta$ , from the total porosity of the felt), we arrive at the expression:

$$\varepsilon_{\text{H}_2\text{O}}^j = \varepsilon_{\text{H}_2\text{O}}^{j-1} \cdot \frac{1}{1 + \Lambda_{jk}} \quad (\text{A17})$$

where:

$\varepsilon_{\text{H}_2\text{O}}^{j-1}$  volumetric fraction of water in the reel at the entrance of press  $j$   
 $\varepsilon_{\text{H}_2\text{O}}^j$  volumetric fraction of water at the exit of press  $j$  and entrance of press  $j+1$   
 $\Lambda_{jk}$  extraction factor of the processed reel  $i$  in interval  $k$  in press  $j$

The extraction factor  $\Lambda_{jk}$  relates parameters of the felt and the processed reel:

$$\Lambda_{jk} = \frac{v_f \cdot e_{f,j} \cdot \phi_j}{v_p \cdot e_p \cdot \phi_p} \cdot k \cdot (1 - \beta) \quad (\text{A18})$$

Using equations (A11) and (A15), it follows that:

$$\Lambda_{jk} = \Lambda_{jk}^0 \cdot \frac{(1 - \phi_{0,j}) \cdot \exp(-\alpha_j \cdot t)}{[1 - \phi_{0,j} \cdot \exp(-\alpha_j \cdot t)]} \quad (\text{A19})$$

$$\Lambda_{jk}^0 = \sum_{i=1}^N \lambda_{ij} \cdot x_{ik} \quad (\text{A20})$$

$$\lambda_{ij} = \frac{v_f \cdot e_{f,j}^0 \cdot \phi_{0,j}}{v_p \cdot e_p \cdot \phi_p} \cdot k \cdot (1 - \beta) \quad (\text{A21})$$

Observe that  $\lambda_{ij}$  is a constant that depends on each felt  $j$  and each processed reel  $i$ , grouping together properties of each paper reel  $i$  ( $e_p$ ,  $\phi_p$ ), each new felt  $j$  ( $e_{f,j}^0$ ,  $\phi_{0,j}$ ) and both ( $k$ ), as well as operating conditions of the pressing machine ( $v_f$ ,  $v_p$ ,  $\beta$ ). However, it is considered that  $v_f = v_p$ , since felt and paper are pressed together in the nip, without slipping.

The parameters  $\lambda_{ij}$  have to be determined experimentally from the ratio of exit and inlet volumetric fractions of water for each press machine, for example from the initial operation of the pressing (observe that for  $t = 0$  we have that  $\Lambda_{jk} = \Lambda_{jk}^0$ ). It is not necessary to determine the other parameters individually ( $k, \beta$ , etc), since all of them are grouped into one parameter ( $\lambda_{ij}$ ) for each reel  $i$  and each felt  $j$ .

In this work, the case of four pressing machines is considered, so that:

$$\varepsilon_{\text{H}_2\text{O}}^1 = \varepsilon_{\text{H}_2\text{O}}^0 \cdot \frac{1}{1 + \Lambda_{1k}} \quad (\text{A22})$$

$$\varepsilon_{\text{H}_2\text{O}}^2 = \varepsilon_{\text{H}_2\text{O}}^1 \cdot \frac{1}{1 + \Lambda_{2k}} \quad (\text{A23})$$

$$\varepsilon_{\text{H}_2\text{O}}^3 = \varepsilon_{\text{H}_2\text{O}}^2 \cdot \frac{1}{1 + \Lambda_{3k}} \quad (\text{A24})$$

$$\varepsilon_{\text{H}_2\text{O}}^4 = \varepsilon_{\text{H}_2\text{O}}^3 \cdot \frac{1}{1 + \Lambda_{4k}} \quad (\text{A25})$$

Therefore:

$$\varepsilon_{\text{H}_2\text{O}}^s = \varepsilon_{\text{H}_2\text{O}}^e \cdot \frac{1}{1 + \Lambda_{1k}} \cdot \frac{1}{1 + \Lambda_{2k}} \cdot \frac{1}{1 + \Lambda_{3k}} \cdot \frac{1}{1 + \Lambda_{4k}} \quad (\text{A26})$$

From Equations (A2) and (A6), and using (A26), we then have:

$$\dot{m}_k^{\text{sp}} = \dot{m}_k^e \cdot \prod_{j=1}^4 \frac{1}{[1 + \Lambda_{jk}]} \quad (\text{A27})$$

In order to apply equation (A1), it is necessary to perform a mathematical integration over Equation (A27). Since the time of processing a reel is much smaller than the time of the useful life of a felt, a numerical integration in each time interval can be done by using the average point in the interval:

$$\int_a^b f(t) \cdot dt \cong f\left(\frac{a+b}{2}\right) \cdot (b-a) \quad (\text{A28})$$

In this way, from (A7), (A27) and (A28), the equation for the water mass that leaves the sector of pressing in the time  $t_k$  in interval  $k$  is given by:

$$\dot{m}_k^{\text{sp}} = \dot{m}_k^e \cdot \prod_{j=1}^4 \frac{1}{[1 + \bar{\Lambda}_{jk}]} \quad k = 1, \dots, N \quad (\text{A29})$$

where the extraction factor  $\bar{\Lambda}_{jk}$  depends on the time of the felt, as follows:

$$\bar{\Lambda}_{jk} = \Lambda_{jk}^0 \cdot \frac{(1 - \phi_{0,j}) \cdot \exp(-\alpha_j \cdot \bar{\tau}_{jk})}{[1 - \phi_{0,j} \cdot \exp(-\alpha_j \cdot \bar{\tau}_{jk})]} \quad (\text{A30})$$

$$\bar{\tau}_{jk} = \theta_{jk} + 0.5 \cdot t_k \quad (\text{A31})$$

where  $\Lambda_{jk}^0$  is the extraction factor of a new felt  $j$  for each reel  $i$  processed at interval  $k$  (equations A20 and A21),  $\alpha_j$  is the coefficient of reduction of the useful life of a felt, and  $\phi_{0,j}$  is the initial porosity of the felt  $j$  ( $\phi_{0,j} = 1 - \xi_{0,j}$ ).